

Construction of higher dimensional charged gravastars: a survey report

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Abstract

We explore possibility to find out a new model of gravastars in the extended D -dimensional Einstein-Maxwell spacetime. The class of solutions as obtained by Mazur and Mottola of a neutral gravastar [1,2] have been observed as an alternative to D -dimensional versions of the Schwarzschild-Tangherlini black hole. To tackle the spherical system in a convenient way we have configured that the gravastar consists of three distinct regions with different equations of state as follows: [I] Interior region $0 \leq r < r_1$, $\rho = -p$, [II] Thin shell region $r_1 \leq r < r_2$, $\rho = p$, and [III] Exterior region $r_2 < r$, $\rho = p = 0$. The outer region of this gravastar model therefore corresponds to a higher dimensional Reissner-Nordström black hole. In connection to this junction conditions are provided and therefore we have formulated mass and the related Equation of State of the gravastar. It has been shown that the model satisfies all the requirements of the physical features. However, overall observational survey of the results also provide probable indication of non-applicability of higher dimensional approach for construction of a gravastar with or without charge from an ordinary 4-dimensional seed as far as physical ground is concerned.

Key words: General relativity; Equation of State; Gravastar

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1 Introduction

A few years ago Mazur and Mottola [1,2] have proposed a new solution for the endpoint of a gravitational collapse. By extending the concept of Bose-Einstein condensation to gravitational systems they constructed a cold compact object with an interior de Sitter condensate phase and an exterior Schwarzschild geometry. These are separated by a phase boundary with a small but finite thickness $r_2 - r_1 = \delta$ of the thin shell under the equation of state (EOS) as follows:

I. Interior: $0 \leq r < r_1$, with EOS $p = -\rho$,

II. Shell: $r_1 < r < r_2$, with EOS $p = +\rho$,

III. Exterior: $r_2 < r$, with EOS $p = \rho = 0$.

Here the presence of matter on the shell is required to achieve the stability of the systems under expansion by exerting an inward force to balance the repulsion from within. These types of *gravitationally vacuum stars* were termed as *gravastars*. Thereafter several scientists have been studied these models under different viewpoints and have opened a new arena of research as an alternative to *Black Holes* [3,4,5,6,7,8,9,10,11,12,13,14,15].

Very recently, a charged (3+1)-dimensional gravastar admitting conformal motion has proposed [16] in the frame work of Mazur and Mottola model [1,2]. In this work the authors provide an alternative to static black holes. However, energy density here is found to diverge in the interior region of the gravastar. This actually scales like an inverse second power of its radius and eventually makes the model singular at $r = 0$. In one of the solutions it is shown that the total gravitational mass vanishes for vanishing charge and turns the total gravitational mass into an electromagnetic mass under certain conditions. Another work on charged gravastar can be found out in Ref. [17] following and admitting conformal motion of Ref. [16] with higher dimensional space-time.

The present study actually is a continuation of the above mentioned work of Usmani et al. [16] on gravastar and its generalization to the higher dimensional space-time, however without admitting conformal motion. Our main motivation here is to construct gravastars in the Einstein-Maxwell geometry and see the higher dimensional effects, if any. Therefore this investigation is also extension of the work of Bhar [17] without admitting conformal motion and that of Rahaman et al. [18] with charge where originally higher dimensional gravastar has been studied.

The plan of the present investigation is as follows: In Sect. 2 the Einstein-Maxwell space-time geometry has been provided as the background of the

study whereas in Sect. 3 we discuss the Interior space-time, Exterior space-time and Thin shell cases of the gravastars with their respective solutions. We explore physical features of the models, viz. proper length, energy and entropy, through the respective graphical plots in Sect. 4. The related junction conditions are provided in Sect. 5. There after the formulation of mass and the Equation of State of the gravastar are given in Sect. 6 and 7 respectively. At the end in Sect. 8 we provide some critically discussed concluding remarks.

2 The Einstein-Maxwell space-time geometry

For higher dimensional gravastar, we assume a D -dimensional spacetime with the structure $R^1 X S^1 X S^d (d = D - 2)$, where S^1 is the range of the radial coordinate r and R^1 is the time axis. For this purpose, let us consider a static spherically symmetric metric in $D = d + 2$ dimension as

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\Omega_d^2. \quad (1)$$

The notation, $d\Omega_d^2$ is a linear element on a d -dimensional unit sphere, parametrized by the angles $\phi_1, \phi_2, \dots, \phi_d$:

$$d\Omega_d^2 = d\phi_d^2 + \sin_2 \phi_d [d\phi_{d-1}^2 + \sin_2 \phi_{d-1} \{d\phi_{d-2}^2 + \dots + \sin_2 \phi_3 (d\phi_2^2 + \sin_2 \phi_2 d\phi_1^2) \dots \}].$$

The Hilbert action coupled to matter is given by

$$I = \int d^D x \sqrt{-g} \left(\frac{R_D}{16\pi G_D} + L_m \right), \quad (2)$$

where R_D is the curvature scalar in D -dimensional spacetime, G_D denotes the D -dimensional Newton constant and L_m is the Lagrangian for matter distribution. We obtain the following Einstein equation by varying the above action with respect to the metric as

$$G_{ij}^D = -8\pi G_D T_{ij}, \quad (3)$$

where G_{ij}^D denotes the Einstein's tensor in D -dimensional spacetime.

The interior of the star is assumed to be perfect fluid type and can be given by

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij}, \quad (4)$$

where, ρ represents the energy density, p is the isotropic pressure, and u^i is the D -velocity of the fluid.

The Einstein field equations for the metric (1), together with the energy-momentum tensor given in Eq. (2), yield

$$-e^{-\lambda} \left[\frac{d(d-1)}{2r^2} - \frac{d\lambda'}{2r} \right] + \frac{d(d-1)}{2r^2} = 8\pi G_D \rho + E^2, \quad (5)$$

$$e^{-\lambda} \left[\frac{d(d-1)}{2r^2} + \frac{d\nu'}{2r} \right] - \frac{d(d-1)}{2r^2} = 8\pi G_D p - E^2, \quad (6)$$

$$\begin{aligned} \frac{e^{-\lambda}}{2} \left[\nu'' - \frac{\lambda'\nu'}{2} + \frac{\nu'^2}{2} - \frac{(d-1)(\lambda' - \nu')}{r} + \frac{(d-1)(d-2)}{r^2} \right] \\ - \frac{(d-1)(d-2)}{2r^2} = 8\pi G_D p + E^2, \end{aligned} \quad (7)$$

where E is the electric field and ‘ r ’ denotes differentiation with respect to the radial parameter r . Here we have assumed $c = 1$ in geometrical unit.

The conservation equation in D -dimensions takes the form

$$\frac{1}{2} (\rho + p) \nu' + p' = \frac{1}{4\pi r^d} (r^d E)'. \quad (8)$$

The charge E is therefore given by

$$(r^d E)' = 4\pi r^d \sigma e^{\lambda/2}. \quad (9)$$

By assuming $\sigma e^{\lambda/2} = \sigma_0 r^m$, m being the polynomial index, we obtain from above Eq. (9)

$$E = \frac{q}{r^d} = 4\pi\sigma_0 \frac{r^{(m+1)}}{(d+m+1)} = A r^{m+1}, \quad (10)$$

where $A = 4\pi\sigma_0/(d+m+1)$.

3 The gravastar models

3.1 Interior space-time

Let us now, following Mazur-Mottola [1], assume the EOS for the interior region in the form

$$p = -\rho. \quad (11)$$

The above EOS is known in the literature as a ‘false vacuum’, ‘degenerate vacuum’, or ‘ ρ -vacuum’ [19,20,21,22]. This type of EOS represents a repulsive pressure, an agent responsible for the accelerating phase of the Universe, and is termed as the Λ -dark energy [23,24,25,26,27]. It is argued by Usmani et al. [16] that a charged gravastar seems to be connected to the dark star [28,29,30].

Using the above EOS, we obtain from Eq. (8)

$$\rho = -p = \frac{\sigma_0 r^{m+1}}{m+1}. \quad (12)$$

Using Eq. (11) in the field equation (5), one gets the solution of λ as given below

$$e^{-\lambda} = 1 - C_1 r^{m+3} - C_2 r^{2(m+2)} + C_3 r^{1-d}, \quad (13)$$

where $C_1 = 16\pi G_D \sigma_0 / d(m+1)(d+m+2)$, $C_2 = 32\pi^2 \sigma_0^2 / d(2m+d+3)(d+m+1)^2$ and C_3 is an integration constant. Since $d > 2$ for dimension higher than four and the solution is regular at $r = 0$, so we demand $C_3 = 0$. Thus essentially we get

$$e^{-\lambda} = 1 - C_1 r^{m+3} - C_2 r^{2(m+2)}. \quad (14)$$

Using Eq. (11) one may obtain from Eqs. (5) and (6), the following relation

$$\ln k = \lambda + \nu, \quad (15)$$

where k is an integration constant.

Thus we have the following interior solutions for the metric potentials λ and ν as follows

$$ke^{-\lambda} = e^\nu = k \left(1 - C_1 r^{m+3} - C_2 r^{2(m+2)} \right). \quad (16)$$

The active gravitational mass $M(r)$ in higher dimensions, can be now calculated as

$$\begin{aligned}
M(r) &= \int_0^{r_1=R} \left[\frac{2\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+1}{2}\right)} \right] r^d \rho dr \\
&= \left[\frac{2\pi^{\frac{d+1}{2}}}{(d+1)\Gamma\left(\frac{d+1}{2}\right)} \right] \left\{ \frac{\sigma_0 R^{d+m+2}}{(m+1)(d+m+2)} \right\}.
\end{aligned} \tag{17}$$

This is the usual gravitating mass for a d -dimensional sphere of radius R and energy density ρ . The space-time metric thus obtained turns out to be free from any central singularity. Another interesting point one can, from Eqs. (10), (12), (16) and (17), easily observe that density, pressure and mass like physical parameters do vanish and the space-time becomes flat for vanishing charge density σ_0 . Therefore, the interior solutions provide electromagnetic mass (EMM) model [31,32,33,34,35,36,37,38,39,40,41,42,43]. This result suggests that the interior de Sitter vacuum of a charged gravastar is essentially an electromagnetic mass model that must generate the gravitational mass [16]. A detailed discussion on this EMM model can be obtained in Ref. [18].

3.2 Exterior space-time

The exterior region defined as EOS $p = \rho = 0$ in higher dimensions is nothing but a generalization of Schwarzschild solution. Following Tangherlini [44] this can be obtained as

$$ds^2 = - \left(1 - \frac{\mu}{r^{d-1}} + \frac{q^2}{r^{2(d-1)}} \right) dt^2 + \left(1 - \frac{\mu}{r^{d-1}} + \frac{q^2}{r^{2(d-1)}} \right)^{-1} dr^2 + d\Omega_d^2. \tag{18}$$

Here $\mu = 16\pi G_D M / \Omega_d$ is the constant of integration with M , the mass of the black hole and Ω_d , the area of a unit d -sphere as $\Omega_d = 2\pi^{(\frac{d+1}{2})} / \Gamma(\frac{d+1}{2})$.

3.3 Intermediate thin shell

Here we assume that the thin shell contains ultra-relativistic fluid of soft quanta and obeys the EOS

$$p = \rho. \tag{19}$$

This relation represents essentially a stiff fluid model as envisioned by Zel'dovich [45] in connection to cold baryonic universe and have been considered by several authors for various situations in cosmology [46,47] as well as in astrophysics [48,49,50].

It is difficult to obtain a general solution of the field equations in the non-vacuum region, i.e. within the shell. We try to find an analytic solution within the thin shell limit, $0 < e^{-\lambda} \equiv h \ll 1$. As an advantage of it, we may set h to be zero to the leading order. Under this approximation, the field Eqs. (5) - (7), with the above EOS, may be recast in the following form

$$\frac{h'}{2r} = \frac{(d-1)}{r^2} - \frac{2E^2}{d}, \quad (20)$$

$$\frac{\nu' h'}{4} + \frac{(d-1)h'}{2r} = -\frac{(d-1)}{r^2} + 2E^2. \quad (21)$$

Integration of Eq. (20) immediately yields

$$h = k_1 + 2(d-1) \ln r - \frac{2A^2 r^{2(m+2)}}{d(m+2)}, \quad (22)$$

where k_1 is an integration constant. The range of r lies within the thickness of the shell $[r_1 = R, r_2 = R + \epsilon]$. We, under the condition $\epsilon \ll 1$, get $E \ll 1$ as $h \ll 1$.

The other metric potential, ν , can be found as

$$\nu = 2 \int \frac{\frac{-d(d-1)}{r} + 2A^2 r^{2(m+1)}(1 + \frac{1}{d})}{(d-1) - \frac{2A^2}{d} r^{2(m+2)}} dr. \quad (23)$$

The result of the above integration has been obtained graphically for different values of d and m as shown in Fig. 1.

The electric field within the shell is assumed to be constant, so we demand that the RHS of Eq. (8) is zero. Also, from the conservation equation and using the same EOS as above, one may obtain

$$p = \rho = \rho_0 e^{-\nu} \quad (24)$$

ρ_0 being an integration constant.

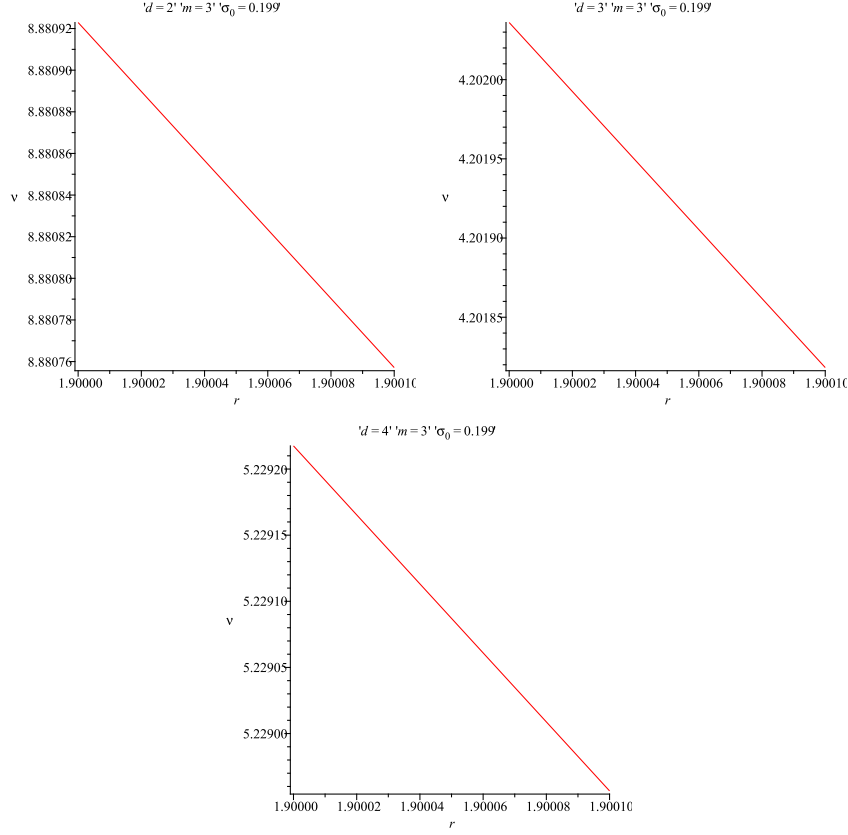


Fig. 1. Variation of ν within the shell against r for different dimensions where the specific legends used are shown in the respective plots

4 Physical features of the models

4.1 Proper length

We consider that matter shell is situated at the surface $r = R$, describing the phase boundary of region I. The thickness of the shell ($\epsilon \ll 1$) is assumed to be very small. Thus the region III joins at the surface $r = R + \epsilon$.

Now, we calculate the proper thickness between two interfaces i.e. of the shell as

$$\ell = \int_R^{R+\epsilon} \sqrt{e^\lambda} dr = \int_R^{R+\epsilon} \frac{dr}{[k_1 + 2(d-1) \ln r - \frac{2A^2 r^{2(m+2)}}{d(m+2)}]^{\frac{1}{2}}}. \quad (25)$$

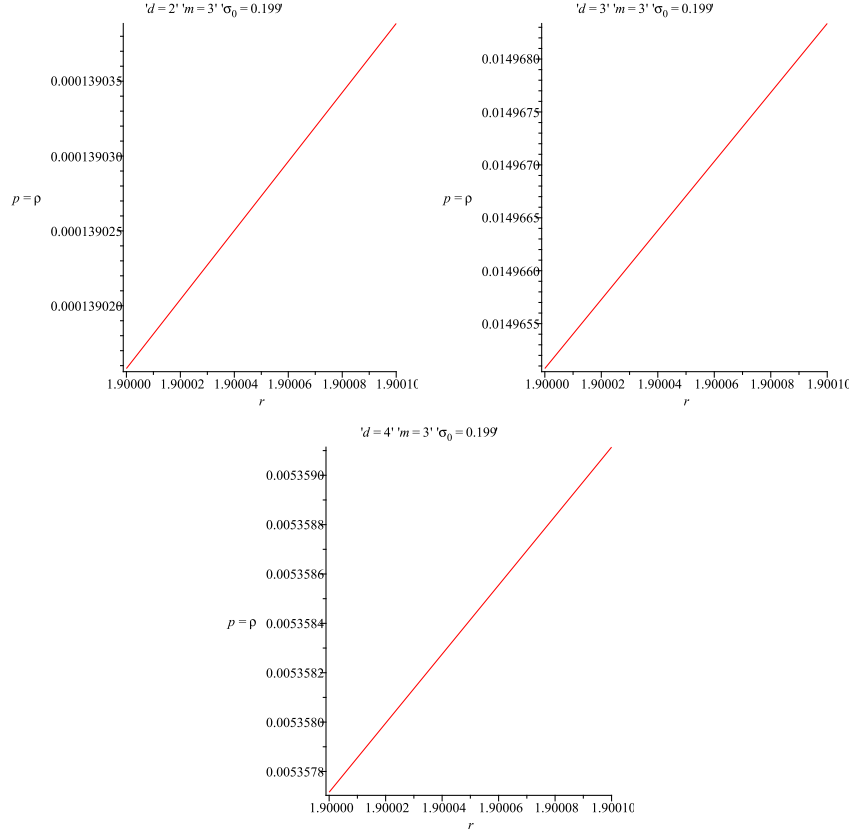


Fig. 2. Variation of the pressure and density of the ultra relativistic matter in the shell against r for different dimensions where the specific legends used are shown in the respective plots

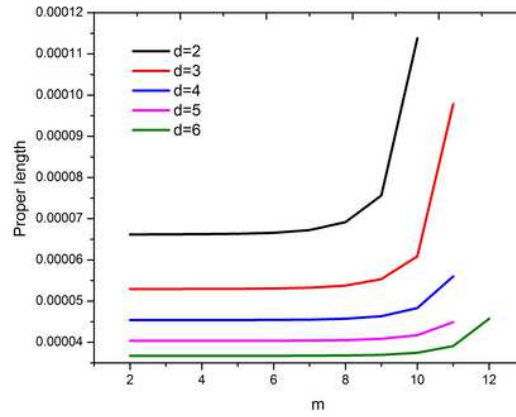


Fig. 3. Variation of the length with m for different dimensions where the specific legends used are shown in the respective plots

4.2 Energy condition

Let us now calculate the energy \tilde{E} within the shell. As the matter density in the shell remains constant, so it demands p' must be zero. but the density will change due to the charge present inside the shell within R to $R + \epsilon$, assuming the matter density $\rho = \rho_c$ we have the energy inside the shell is

$$\tilde{E} = \int_R^{R+\epsilon} \left[\frac{2\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+1}{2}\right)} \right] r^d \left[\rho + \frac{E^2}{8\pi} \right] dr = \left[\frac{2\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+1}{2}\right)} \right] \epsilon \left[\rho_c R^d + \frac{A^2}{8\pi} R^{2(m+1)+d} \right] \quad (26)$$

The energy \tilde{E} within the shell up to first order in ϵ has been considered. As the thickness ϵ of the shell is very small i.e. $\epsilon \ll 1$, so we expand it binomially about R and taking first order of ϵ only. However, we observe that the energy within the shell is not only proportional to ϵ in first order of thickness but also depends on dimension d of the spacetime. In this connection we have drawn several plots of entropy for different values of d and m as shown in Fig. 2 which show physically expected properties of the model of charged gravastar in higher dimension.

4.3 Entropy

We calculate the entropy following Mazur and Mottola prescription [1] as

$$S = \int_R^{R+\epsilon} \left[\frac{2\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+1}{2}\right)} \right] r^d s(r) \sqrt{e^\lambda} dr. \quad (27)$$

Here, $s(r)$ stands for the entropy density of the local temperature $T(r)$, which may be written as

$$s(r) = \frac{\alpha^2 k_B^2 T(r)}{4\pi \hbar^2} = \alpha \left(\frac{k_B}{\hbar} \right) \sqrt{\frac{p}{2\pi}}, \quad (28)$$

where α is a dimensionless constant.

Thus the entropy of the fluid within the shell could be found as

$$S = \int_R^{R+\epsilon} \left[\frac{2\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+1}{2}\right)} \right] \sqrt{\frac{\alpha^2 \rho_0}{2\pi}} \left(\frac{k_B}{\hbar} \right) \frac{r^d dr}{[k_1 + 2(d-1) \ln r - \frac{2A^2 r^{2(m+2)}}{d(m+2)}]^{\frac{1}{2}}}. \quad (29)$$

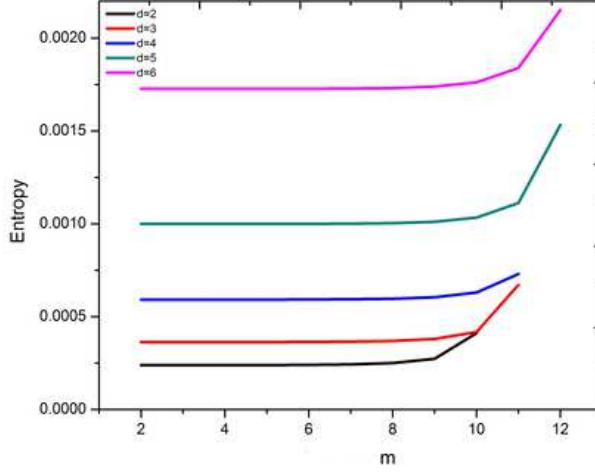


Fig. 4. Variation of entropy with m for different dimensions where the specific legends used are shown in the respective plots

Following Usmani et al. [16] it can be shown that for the thickness of the shell, which is negligibly small compared to its position from the center of the gravastar (i.e., if $\epsilon \ll R$), the entropy is given by $S = (\alpha k_B / 4\pi \hbar G_D) \epsilon R$. The plots of entropy for different values of d and m are obtained in Fig. 4 which show physically valid features for the present model of charged gravastar in higher dimension.

5 Junction Condition

In the gravastar configuration, there are three regions namely, interior region, shell and exterior region. Interior and exterior regions join at the junction interface i.e. at the shell. At the shell, the metric coefficients are continuous but it does not guarantee that their derivatives are also continuous there. Considering this we use the Darmois-Israel [51,52] formation to calculate the surface stresses at the junction interface. The Lanczos equations provide the intrinsic surface stress energy tensor S_{ij} in the following form

$$K_{ij}^{\pm} = -n_{\nu}^{\pm} \left[\frac{\partial^2 X_{\nu}}{\partial \xi^1 \partial \xi^j} + \Gamma_{\alpha\beta}^{\nu} \frac{\partial X^{\alpha}}{\partial \xi^i} \frac{\partial X^{\beta}}{\partial \xi^j} \right] |_S, \quad (30)$$

where the unit normal vector n_{ν}^{\pm} are the defined as

$$n_{\nu}^{\pm} = \pm \left| g^{\alpha\beta} \frac{\partial f}{\partial X^{\alpha}} \frac{\partial f}{\partial X^{\beta}} \right|^{-\frac{1}{2}} \frac{\partial f}{\partial X^{\nu}} \quad (31)$$

with $n^\nu n_\nu = 1$. Following the Lanczos equations one can determine the surface energy density (Σ) and surface pressure $p_{\theta_1} = p_{\theta_2} = \dots = p_{\theta_{d-1}} = p_t$ as

$$\Sigma = -\frac{d}{4\pi R} \left[\sqrt{1 - \frac{\mu}{R^{d-1}} + \frac{q^2}{R^{2(d-1)}}} - \sqrt{1 - C_1 R^{m+3} - C_2 R^{2(m+2)}} \right], \quad (32)$$

$$p_t = \frac{1}{8\pi R} \left[\frac{(d-1) - (d-1)\frac{\mu}{2R^{d-1}}}{\sqrt{1 - \frac{\mu}{R^{d-1}} + \frac{q^2}{R^{2(d-1)}}}} + \frac{C_1(m+3)R^{m+2} + 2C_2(m+2)R^{2m+3}}{\sqrt{1 - C_1 R^{m+3} - C_2 R^{2(m+2)}}} \right]. \quad (33)$$

6 Mass of the gravastar

Now, it is easy to find the mass m_s of the thin shell from the equation

$$m_s = 4\pi R^2 \Sigma = -Rd \left[\sqrt{1 - \frac{\mu}{R^{d-1}} + \frac{q^2}{R^{2(d-1)}}} - \sqrt{1 - C_1 R^{m+3} - C_2 R^{2(m+2)}} \right]. \quad (34)$$

Using Eqs. (35) and (37) we can determine the mass of the gravastar in terms of the mass of the thin shell as

$$\mu = \frac{q^2}{R^{d-1}} + R^{d-1} \left[C_1 R^{m+3} + C_2 R^{2(m+2)} + \frac{m_s}{Rd} \left(2\sqrt{1 - C_1 R^{m+3} - C_2 R^{2(m+2)}} - \frac{m_s}{Rd} \right) \right]. \quad (35)$$

7 Equation of State

Let us assume $p_{\theta_1} = p_{\theta_2} = p_{\theta_3} = \dots = p_t = -\mathcal{P}$, here \mathcal{P} is the surface tension as acts on the fluid of the gravastar.

Then Eqs. (32) and (33) yield

$$\mathcal{P} = \omega(R)\Sigma. \quad (36)$$

Thus the Equation of State parameter ω can be found as

$$\omega(R) = \frac{\left[\frac{(d-1) - (d-1)\frac{\mu}{2R^{d-1}}}{\sqrt{1 - \frac{\mu}{R^{d-1}} + \frac{q^2}{R^{2(d-1)}}}} + \frac{C_1(m+3)R^{m+2} + 2C_2(m+2)R^{2m+3}}{\sqrt{1 - C_1 R^{m+3} - C_2 R^{2(m+2)}}} \right]}{2d \left[\sqrt{1 - \frac{\mu}{R^{d-1}} + \frac{q^2}{R^{2(d-1)}}} - \sqrt{1 - C_1 R^{m+3} - C_2 R^{2(m+2)}} \right]}. \quad (37)$$

8 Discussions and Conclusions

In the present study we have explored some possibilities to find out a new model of gravastars in contrast of the Mazur-Mottola type model of neutral gravastar [1,2], specifically seeking its generalization to: (i) the extended D -dimensional spacetime, and (ii) the Einstein-Maxwell geometry. Under these two considerations a class of solutions and hence some interesting results have been found out which can be observed as an alternative to D -dimensional versions of the Schwarzschild-Tangherlini black hole. Some of the key physical features of the model are as follows:

(i) We have found out all the physical parameters e.g. metric potentials, thickness of the thin shell, energy, entropy etc. and our result matches to the result of Usmani et al. [16] for $d = 2$ and $m = 0$ i.e. for 4-dimensional space-time without any polynomial index. All the plots (Figs. 1 - 4) related to these parameters also suggest validity of physical requirements.

(ii) We have calculated the active gravitational mass of the core in the interior of the gravastar in Eq. (17). This active gravitational mass through the charge density σ provides EMM model such that vanishing electric charge makes the physical system to vanish and the space-time becomes Euclidean flat.

(iii) It is interesting to note that all the solutions are regular at the center $r = 0$ and positive inside the interior region of the gravastar.

As a final comment, we would like to put an important note here regarding overall observational result of the present investigation on gravastar with higher dimensional space-time. As a sample study a comparison between Figs. 1 - 4 shows that though there are quantitative changes in the profile of some physical parameters but qualitatively not much appreciable results can be observed. Especially, a close and keen observation between profiles of plots of Figs. 1 and 2 for different dimensions do not indicate any departure from each other. All these observational surveys are probable indication of non-applicability of higher dimensional approach for construction of a gravastar with or without charge from an ordinary 4-dimensional seed.

Acknowledgments

FR and SR wish to thank the authorities of the Inter-University Centre for Astronomy and Astrophysics, Pune, India for providing the Visiting Associateship under which a part of this work was carried out.

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